NAG Toolbox for MATLAB

d01gy

1 Purpose

d01gy calculates the optimal coefficients, for use by d01gc and d01gd for prime numbers of points.

2 Syntax

```
[vk, ifail] = d01gy(ndim, npts)
```

3 Description

The Korobov procedure Korobov 1963 for calculating the optimal coefficients $a_1, a_2, \ldots, a_n$ for p-point integration over the n-cube $[0,1]^n$ imposes the constraint that

$$a_1 = 1 \qquad \text{and} \qquad a_i = a^{i-1} (\mathrm{mod}\, p), \qquad i = 1, 2, \ldots, n \tag{1}$$

where p is a prime number and a is an adjustable parameter. This parameter is computed to minimize the error in the integral

$$3^n \int_0^1 dx_1 \cdots \int_0^1 dx_n \prod_{i=1}^{n} (1 - 2x_i)^2, \tag{2}$$

when computed using the number theoretic rule, and the resulting coefficients can be shown to fit the Korobov definition of optimality.

The computation for large values of p is extremely time consuming (the number of elementary operations varying as p^2) and there is a practical upper limit to the number of points that can be used. Routine d01gz is computationally more economical in this respect but the associated error is likely to be larger.

4 References

Korobov N M 1963 *Number Theoretic Methods in Approximate Analysis* Fizmatgiz, Moscow

5 Parameters

5.1 Compulsory Input Parameters

1: **ndim – int32 scalar**

n, the number of dimensions of the integral.

Constraint: **ndim** ≥ 1.

2: **npts – int32 scalar**

p, the number of points to be used.

Constraint: **npts** must be a prime number ≥ 5.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **vk**(**ndim**) – **double array**

The n optimal coefficients.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **ndim** < 1.

ifail $= 2$

On entry, **npts** < 5.

ifail $= 3$

On entry, **npts** is not a prime number.

ifail $= 4$

The precision of the machine is insufficient to perform the computation exactly. Try a smaller value of **npts**, or use an implementation of higher precision.

7 Accuracy

The optimal coefficients are returned as exact integers (though stored in a real array).

8 Further Comments

The time taken is approximately proportional to p^2 (see Section 3).

9 Example

```
ndim = int32(4);
npts = int32(631);
[vk, ifail] = d01gy(ndim, npts)
```

```
vk =
     1
   198
    82
   461
ifail =
           0
```